FE Review - Kinetics & Dynamics

Kinematics

Kinematics- The study of the geometry of motion given a position vector $\bar{r}(t)$

$$\begin{aligned}
\text{The velocity vector } \bar{v}(t) &= \frac{d\bar{r}}{dt} \\
\text{The acceleration vector } \bar{a}(t) &= \frac{d\bar{v}}{dt} = \frac{d^2\bar{r}}{dt^2}
\end{aligned}$$

Coordinate Systems

The kinetics (and dynamics) of an object must be described relative to a coordinate system.

Rectangular Coordinates

$$\begin{aligned}
\bar{r} &= x\,\hat{i} + y\,\hat{j} + z\,\hat{k} \\
\bar{v} &= \frac{dx}{dt}\,\hat{i} + \frac{dy}{dt}\,\hat{j} + \frac{dz}{dt}\,\hat{k} \\
&= \dot{x}\,\hat{i} + \dot{y}\,\hat{j} + \dot{z}\,\hat{k} \\
\bar{a} &= \ddot{x}\,\hat{i} + \ddot{y}\,\hat{j} + \ddot{z}\,\hat{k}
\end{aligned}$$

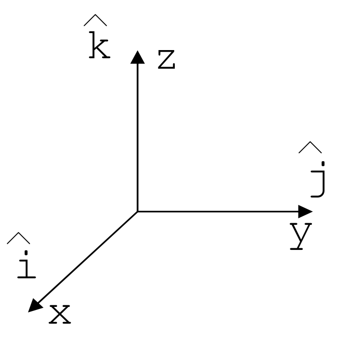


Figure 1: Rectangular coordinates

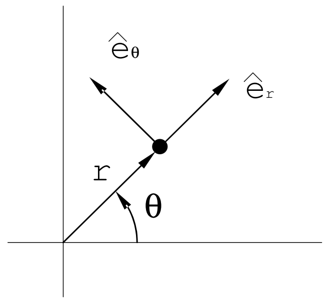


Figure 2: Polar coordinates

Polar Coordinates

$$\begin{aligned} \bar{r} &= r\,\hat{e}_r \\ \bar{v} &= \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta \\ \bar{a} &= (\ddot{r} - r\dot{\theta}^2)\,\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{e}_\theta \\ \text{where } r &: \text{radial position} \\ \theta &: \text{angular position} \\ \dot{r} &: \text{radial velocity} \\ \ddot{r} &: \text{radial acceleration} \\ \dot{\theta} &: \text{angular velocity} \\ \ddot{\theta} &: \text{angular acceleration} \end{aligned} \tag{1}$$

Normal-Tangent Coordinates

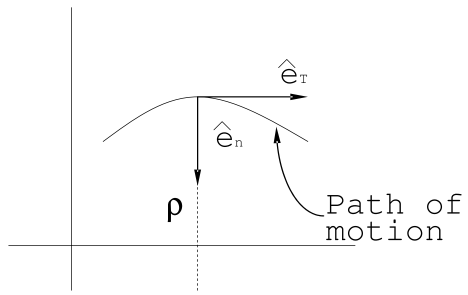


Figure 3: Normal-Tangent coordinates

$$\begin{aligned} \bar{v} &= v\,\hat{e}_\tau \\ \bar{a} &= \frac{dv}{dt}\,\hat{e}_\tau + \frac{v^2}{\rho}\,\hat{e}_n \\ \text{where } v &: \text{tangential velocity} \\ \frac{dv}{dt} &: \text{rate of change of the tangential velocity} \\ \rho &: \text{radius of curvature.} \end{aligned}$$

Motions

Straight Line Motion

Constant Acceleration (with initial velocity v_o and initial position s_o)

$$\frac{dv}{dt} = a_o \quad \rightarrow \quad v = \int_{t_o}^{t} a_o \, dt \quad \rightarrow \quad v = v_o + a_o t$$
$$\frac{ds}{dt} = v \quad \rightarrow \quad s = \int_{t_o}^{t} (v_o + a_o \tau) \, d\tau \quad \rightarrow \quad s = s_o + v_o t + \frac{1}{2} a_o t^2$$

Projectile Motion

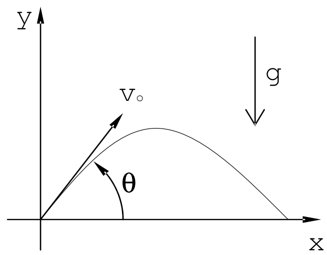


Figure 4: Projectile Motion

Acceleration in the x direction

$$a_x = 0$$
$$v_x = v_o \cos\theta$$
$$x = x_o + v_o t \cos\theta$$

Acceleration in the y direction

$$a_y = -g$$
$$v_y = v_o \sin\theta - gt$$
$$y = y_o + v_o t \sin\theta - \frac{gt^2}{2}$$

Plane Circular Motion

$$r = \text{constant radius}$$
$$\dot{\theta} = \omega = \text{angular velocity}$$
$$\alpha = \dot{\omega} = \text{angular acceleration}$$

Tangential acceleration : $a_\tau = \alpha r$

Normal acceleration : $a_n = r\omega^2$

Tangential velocity : $v_\tau = \omega r$

Position (arc length) : $s = \theta r$

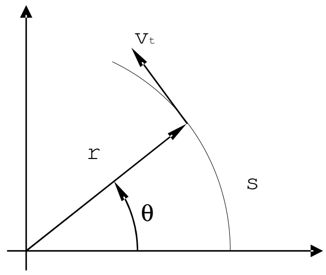


Figure 5: Plane Circular Motion

Newton's Laws of Motion

(1) An object moving in a straight line at constant velocity will remain in motion unless acted on by an external force. An object at rest will remain at rest unless acted on by an external force.

(2) The net external force acting on an object is equal to the rate of change of linear momentum, i.e,

$$\begin{aligned} \bar{F} &= \frac{d}{dt}(m\bar{v}) \\ \text{where } \bar{F} &: \text{net external force} \\ m\bar{v} &: \text{linear momentum} \end{aligned}$$

For constant mass,

$$\bar{F} = m\frac{d\bar{v}}{dt} = m\bar{a}$$

(3) For two object in contact the mutual force on interaction is equal and opposite.

One Dimensional Motion

$$\begin{aligned} F &= ma \rightarrow a(t) = F(t)/m \\ v(t) &= v_o + \int_{t_o}^{t} (F(\tau)/m)\, d\tau \\ x(t) &= x_o + v_o t + \int_{t_o}^{t} v(\tau)\, d\tau \end{aligned}$$

Impulse and Momentum

The impulse of a force is equal to the change in linear momentum

$$\bar{I} = \int_{t_0}^{t_1} \bar{F}\,dt = m\bar{v}(t_1) - m\bar{v}(t_0)$$

If the net external force is zero the linear momentum is conserved

$$\begin{aligned} \bar{I} = \int_{t_0}^{t_1} \bar{F}\,dt = \quad & 0 && = m\bar{v}(t_1) - m\bar{v}(t_0) \\ m\bar{v}(t_1) & = && m\bar{v}(t_0) \end{aligned}$$

This called the 'Principle of Conservation of Linear Momentum'.

Work and Energy

⋄ Work done by a force,

$$W = \int \bar{F}\,d\bar{r}$$

⋄ Work done by a force in moving an object from point 1 to point 2 is the change in kinetic energy

$$\begin{aligned} W &= T_2 - T_1 \\ &= \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \end{aligned}$$

⋄ Work done by a conservative force in moving an object from point 1 to point 2 is

$$W = U_1 - U_2$$

where U_1 is the potential energy at point 1 and U_2 is the potential energy at point 2.

Principle of Conservation of Energy

If all the forces acting on a system are conservative then the total energy is constant, i.e.,

$$E = \underbrace{T_1 + U_1}_{Total\ energy\ at\ 1} = \underbrace{T_2 + U_2}_{Total\ energy\ at\ 2}$$

Potential energy due to gravity

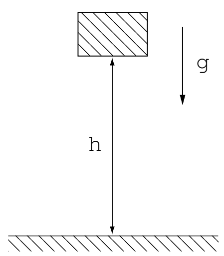


Figure 6: Potential energy due to gravity

$$U_{gravity} = mgh$$

Potential energy due to a spring

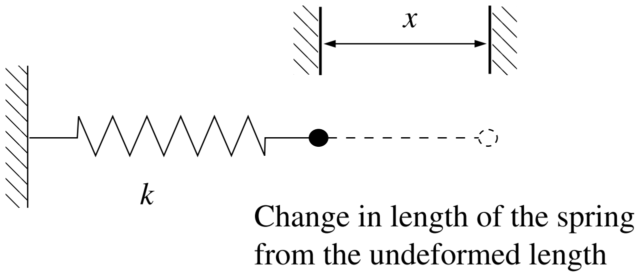


Figure 7: Potential energy due to a spring

$$U_{spring} = \frac{1}{2}kx^2$$

Power

Power is the rate at which work is done

$$P = \frac{dW}{dt}$$

Impact

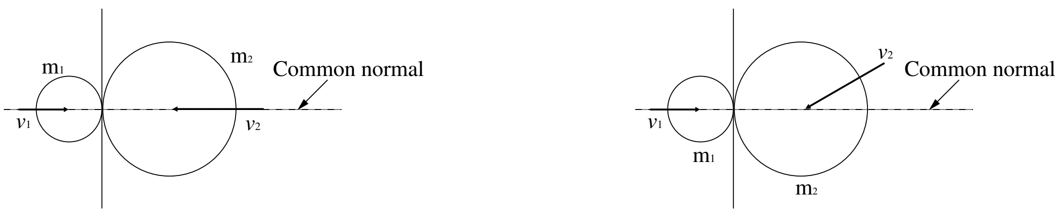


Figure 8: Direct Impact and Oblique Impact

Direct Impact

All velocities before and after impact are along the common normal.

Oblique Impact

There is a velocity component along the common tangent.

Newton's Collision Rule

$$\underbrace{v'_{1_n} - v'_{2_n}}_{relative\ veloctiy\ along\ the\ common\ normal\ after\ impact} = \underbrace{-e(v_{1_n} - v_{2_n})}_{relative\ velocity\ along\ the\ common\ normal\ before\ impact}$$

where e is coefficient of restitution.

$$\underbrace{m_1\bar{v}_1' + m_2\bar{v}_2'}_{linear\ momentum\ after\ impact} = \underbrace{m_1\bar{v}_1 + m_2\bar{v}_2}_{linear\ momentum\ before\ impact}$$

Plane Motion of a Rigid Body

$$\begin{aligned} F_x &= ma_x \\ F_y &= ma_y \\ M_c &= I_c\alpha \end{aligned}$$

M_c - net external moment about the center of mass.
I_c - Mass moment of inertia about the center of mass.
α - Angular acceleration.

Rotation about a fixed point

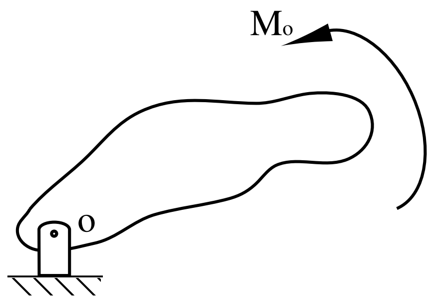


Figure 9: Rotation about a fixed point

$$M_o = I_o\alpha$$

where I_o is a moment of inertia about point O.

Kinetic energy of a plane rigid body

For general motion,

$$T = \frac{1}{2}m(v_x^2 + v_y^2) + \frac{1}{2}I_c\omega^2$$

If the rotation is about a fixed point, then

$$T = \frac{1}{2}I_o\omega^2$$

Free Vibration

The equation of motion for the system shown here is

$$m\ddot{x} + kx = 0.$$

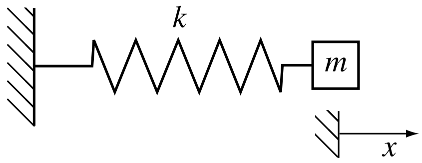


Figure 10: Free vibration

The solution to this differential equation is

$$x(t) = C_1 \cos \omega t + C_2 \sin \omega t,$$

where C_1, and C_2 are constants that depend on the initial conditions, and

$$\omega = \sqrt{\frac{k}{m}},$$

is called the natural frequency. The period of oscillation is given by

$$T = \frac{2\pi}{\omega}.$$

If $x(0) = x_0$, $\dot{x}(0) = v_0$, then

$$x(t) = x_0 \cos \omega t + \frac{v_0}{\omega} \sin \omega t.$$

If $x(0) = x_0$, $\dot{x}(0) = 0$, then

$$x(t) = x_0 \cos \omega t.$$

Torsional Vibration

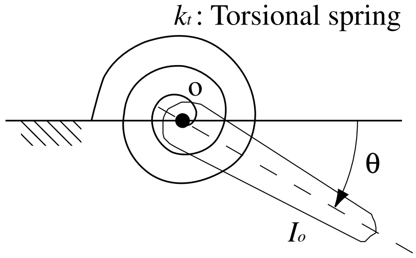


Figure 11: Torsional vibration

In this case the equation of motion is given by

$$\ddot{\theta} + \omega^2 \theta = 0,$$

where

$$\omega = \sqrt{\frac{k_t}{I_0}},$$

is the natural frequency. Here, k_t is the torsional spring stiffness, and I_o is the moment of inertia about O.

Problems

1. The velocity of a particle at time t is

$$v(t) = 12t^4 + 7/t$$

what is the total distance travelled between $t = 0.2$ and $t = 0.3$?

$$\begin{aligned} s &= \int_{t_1}^{t_2} v dt = \int_{0.2}^{0.3} (12t^4 + 7/t) dt \\ &= \frac{12}{5} t^5 + 7 \ln t \,|_{0.2}^{0.3} \\ &= \left[\frac{12}{5}(0.3)^5 + 7\ln(0.3)\right] - \left[\frac{12}{5}(0.2)^5 + 7\ln(0.2)\right] \\ &= 2.84. \end{aligned}$$

2. How far will an object under earth's gravity drop in 13 s, starting from rest and neglecting air friction?

$$\begin{aligned} s &= s_o + v_o t + \frac{a_o}{2} t^2 = \frac{1}{2}(9.81\,\frac{\mathrm{m}}{\mathrm{s}^2})(13\,\mathrm{s})^2 \\ &= 829\,\mathrm{m}. \end{aligned}$$

3. The position of an object is given by

$$s = 4t^3 + 2t^2 - t + 3$$

What is the acceleration at $t = 2$?

$$\begin{aligned} v &= \frac{ds}{dt} = 12t^2 + 4t - 1 \\ a &= \frac{dv}{dt} = 24t + 4 \\ a(2) &= 52. \end{aligned}$$

4. An object weights 2 pounds is travelling in a circular path of radius 5 feet at a constant speed of 10 feet per second. The acceleration of the object is most nearly:

   Since the object is moving in a circular path and the speed along the path is given we can use the normal-tangent coordinate system to get

$$a_\tau = \frac{dv}{dt}, \quad a_n = \frac{v^2}{\rho}.$$

   Since $v = 10\,\mathrm{ft/s}$ is constant we get $a_\tau = 0$. Using $\rho = 5\,\mathrm{ft}$ gives

$$a_n = \frac{(10\,\mathrm{ft/s})^2}{5\,\mathrm{ft}} = 20\,\frac{\mathrm{ft}}{\mathrm{s}^2}.$$

5. A projectile is launched from a level plane at 30° from the horizontal with an initial velocity of 1250 m/s. (a) What is the maximum height above the plane the projectile will reach? (b) What is the maximum range of the projectile at the maximum range $y = 0$?

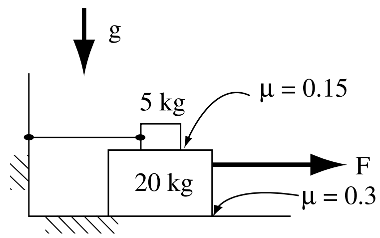
(a)

$$\begin{aligned} v_y &= v_o \sin\theta - gt = 0 \rightarrow t = \frac{v_o}{g}\sin\theta \\ y &= \frac{v_o^2 \sin^2\theta}{g} - \frac{v_o^2 \sin^2\theta}{2g} = \frac{v_o^2 \sin^2\theta}{2g} \\ &= \frac{(1250\,\frac{\text{m}}{\text{s}})^2(\sin^2 30^\circ)}{(2)(9.81\,\frac{\text{m}}{\text{s}^2})(1000\,\frac{\text{m}}{\text{km}})} = 19.9\,\text{km}. \end{aligned}$$

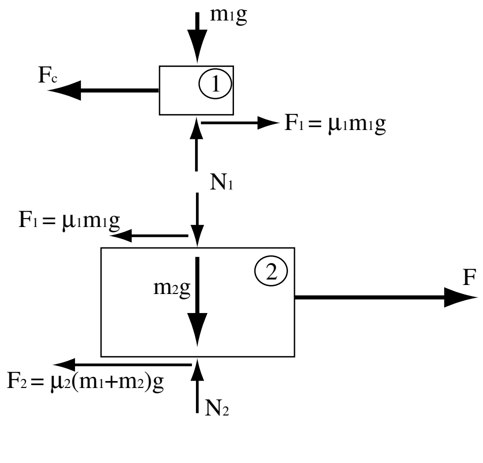
(b)

$$\begin{aligned} y &= 0 \rightarrow t(v_o \sin\theta - gt/2) = 0 \\ \text{therefore} \quad & t = \frac{2v_o \sin\theta}{g} \\ x &= \frac{(v_o)(2v_o)\sin\theta\cos\theta}{g} = \frac{v_o^2 \sin 2\theta}{g} \\ &= \frac{(1250\,\frac{\text{m}}{\text{s}})^2 \sin 60^\circ}{9.81\,\frac{\text{m}}{\text{s}^2}} = 137938\,\text{m} \doteq 138\,\text{km}. \end{aligned}$$

6. The force F is gradually increased until the 20 kg block begins moving to the right. The 5 kg block is prevented from moving by a cord. What is the minimum force F for which movement is possible?



The free body diagram for thw system is as follows,



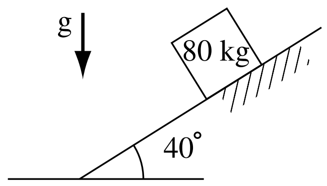
Since body 1 is in equilibrium we have

$$F_c = F_1 = \mu_1 N_1 = \mu_1 m_1 g = (0.15)(5\,\text{kg})(9.81\,\text{m/s}^2) = 7.36\,\text{N}.$$

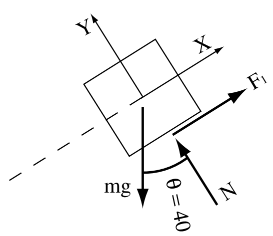
If body 2 is in equilibrium then,

$$\begin{aligned} F &= F_1 + F_2 \\ &= 7.36 + (\mu_2(m_1 + m_2)g) \\ &= 7.36 + (0.3)(25\,\text{kg})(9.81\,\text{m/s}^2) \\ &= 80.9\,\text{N}. \end{aligned}$$

7. What is the frictional force between the 80 kg block and the ramp. Given $\mu_s = 0.2$, and $\mu_f = 0.15$.



The free body diagram for the system is shown below.



Summing the forces along the Y-axis gives,

$$\sum F_Y = 0 \quad \rightarrow \quad N = mg\cos\theta$$
$$N = (80)(9.81\,\frac{\text{m}}{\text{s}^2})\cos 40^\circ = 601.2\,\text{N}.$$

First assume that the system is in static equilbrium. Then, the static friction force along the X-axis satisfies

$$F_{1static} = \mu_s N = (0.2)(601.2) = 120.2\,\text{N}.$$

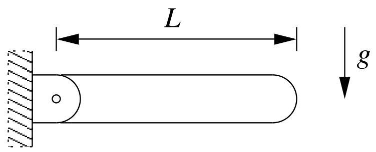
The force due to gravity along the X-axis is

$$F_x = mg\sin\theta = (80)(9.81\,\frac{\text{m}}{\text{s}^2})\sin 40^\circ = 504.5\,\text{N}.$$

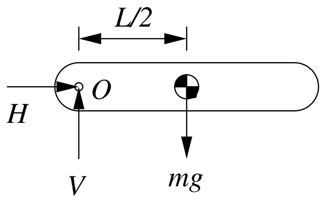
Since, F_x is greater than $F_{1static}$ we can conclude that the block is sliding. Thus, the sliding friction force is given by

$$F_f = \mu_f N = (0.15)(601.2\,\text{N}) = 90.17\,\text{N}.$$

8. The slender homogeneous rod shown below has just been released from rest. If g is the acceleration due to gravity, the magnitude of the angular acceleration of the rod is:



The free body diagram below shows the reaction forces at the pivot (i.e., H and V), as well as the weight of the rod.



Free body diagram

Since the pivot O is a fixed point on the rod the equation of motion is

$$M_O = I_O\alpha,$$

where M_O is the sum of the moments about O, I_O is the moment of inertia about O, and α is the angular acceleration. Here,

$$M_O = mg\frac{L}{2}, \text{ and } I_O = \frac{1}{3}mL^2.$$

Therefore,

$$\begin{aligned} mg\frac{L}{2} &= \frac{1}{3}mL^2\alpha \\ \alpha &= \frac{3g}{2L}. \end{aligned}$$

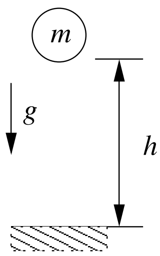
9. A rocket is moving through a vacuum. It changes its velocity from $9,020\,\text{m/s}$ to $5,100\,\text{m/s}$ in $48\,\text{s}$. How much power is required to accomplish this if the rocket mass is $213,000\,\text{kg}$?

$$\begin{aligned} P &= \frac{dW}{dt} = \frac{\Delta E}{\Delta t} = \frac{T_1 - T_2}{\Delta t} \\ T_1 &= \frac{1}{2}mv_1^2, \quad T_2 = \frac{1}{2}mv_2^2 \\ P &= \frac{\frac{1}{2}(213000\,\text{kg})[(9020)^2 - (5100)^2]\frac{\text{m}^2}{\text{s}^2}}{48\,\text{s}} \\ &= 122.8 \times 10^9\,\text{W} \doteq 123\,\text{GW}. \end{aligned}$$

10. A $60\,\text{kg}$ ball is dropped from a height at $48\,\text{m}$ above a table. (a) What is the velocity of the ball just before impact? (b) If $e = 0.9$, what is the kinetic energy of the ball immediately after impact?

(a) Applying the Principle of Conservation of Energy gives,

$$\begin{aligned} T_1 + U_1 &= T_2 + U_2 \\ mgh &= \frac{1}{2}mv_2^2 \end{aligned}$$



$$v_2 = \sqrt{2gh} = \sqrt{(2)(9.81\,\frac{\text{m}}{\text{s}^2})(48\,\text{m})} = 30.7\,\text{m/s}.$$

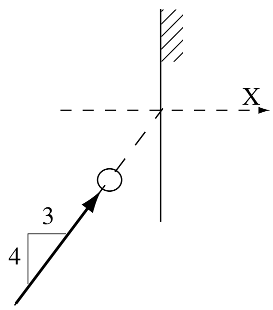
(b) The subscript of 'b' is related the ball, and the subscript of 'g' is related the ground. From Newton's collision rule we have

$$v_b' - v_g' = -e(v_b - v_g).$$

Therefore, the velocity of the ball after impact is $v_b' = -ev_b = -(0.9)(30.7)\,\text{m/s}$. Hence the kinetic energy after impact is

$$T = \frac{1}{2}m\,{v_b'}^2 = (\frac{1}{2})(60)(-(0.9)(30.7))^2 = 22900\,\text{J} = 22.9\,\text{kJ}.$$

11. What is the component of velocity perpendicular to the wall after impact, if $e = 0.8$ and the velocity before impact has magnitude 50 m/s?



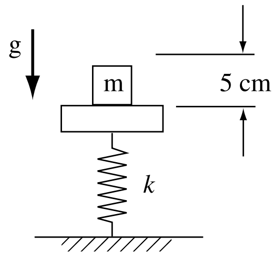
The X direction defines the common normal therefore, Newton's collision rule gives

$$\begin{aligned} v_x' &= -ev_x \\ v_x &= 50\,\frac{\text{m}}{\text{s}}\frac{3}{\sqrt{3^2+4^2}} \\ v_x' &= -24\,\text{m/s}. \end{aligned}$$

12. A spring has a spring constant $k = 10\,\text{N/cm}$. It is compressed 5 cm. The spring is released and pushes against a free projectile with a mass 1 kg. What is the projectile velocity immediately after losing contact with the spring?

Apply the Principle of Conservation of Energy to get,

$$\begin{aligned} T_1 + U_1 &= T_2 + U_2 \\ \frac{1}{2}kx^2 &= \frac{1}{2}mv^2 \\ v &= x\sqrt{\frac{k}{m}} = 5\,\text{cm}\frac{1\,\text{m}}{100\,\text{cm}}\sqrt{\frac{10\,\frac{\text{N}}{\text{cm}}\,100\,\frac{\text{cm}}{\text{m}}}{1\,\text{kg}}} = 1.5\,\text{m/s}. \end{aligned}$$

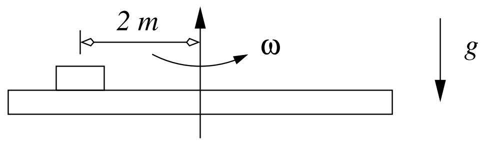


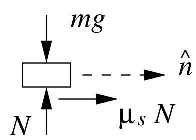
13. A projectile is launched at 52° from the horizontal with an initial velocity of 3600 m/s. If the mass of the projectile is 32 kg what is the total kinetic energy and potential energy when $t = 13\,\text{s}$? Neglect all forms of the friction.

    Apply the Principle of Conservation of energy to get

$$
\begin{aligned}
E &= T_1 + U_1 = T_2 + U_2 \\
T_1 &= T_2 + U_2 = \frac{1}{2}mv^2 = \frac{1}{2}(32\,\text{kg})(3600\,\frac{\text{m}}{\text{s}})^2 \frac{1\,\text{MJ}}{10^6\,\frac{\text{kg m}^2}{\text{s}^2}} = 207\,\text{MJ} = E.
\end{aligned}
$$

14. The block rests at a distance of 2 m from the center of the rotating platform. If the coefficient of static friction between the block and the platform is $\mu_s = 0.3$, determine the maximum speed the block can attain before it begins to slip.





Freebody diagram

In the freebody diagram $\hat{n}$ denotes the normal direction of the normal-tangent coordinate system. (The tangent is directed out of the paper.) Applying Newton's second law in the z and $\hat{n}$ directions gives

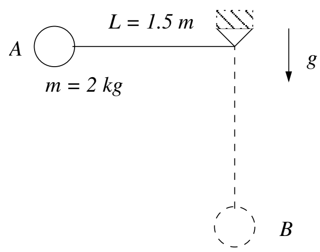
$$\sum F_z = ma_z \rightarrow N - mg = 0 \rightarrow N = mg \qquad (i)$$

$$\sum F_{\hat{n}} = ma_{\hat{n}} = m\frac{v^2}{\rho} \qquad (ii)$$

In equation (ii) v is the velocity of the block tangent to the path of motion, and ρ is the radius of curvature. Also, $F_{\hat{n}} = \mu_s N$ therefore, equation (ii) yields

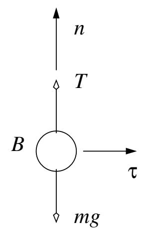
$$
\begin{aligned}
\mu_s N &= m\frac{v^2}{\rho} \\
(0.3)(m\,9.81) &= m\frac{v^2}{2} \\
&\rightarrow \quad v = \sqrt{(2)(0.3)(9.81)} = 2.43\ \text{m/s}
\end{aligned}
$$

15. A pendulum bob is released from rest when it is at A. Determine the speed of the bob and the tension in the cord when the bob passes through its lowest position B. The length of the cord is 1.5 m, and the bob has mass 2 kg.



Using the lowest point, B, as our reference we can apply the Prinicple of Conservation of Energy at points A and B to get

$$
\begin{aligned}
T_A + U_A &= T_B + U_B \\
0 + mgh_A &= \frac{1}{2} m v_B^2 + 0 \\
0 + (2)(9.81)(1.5) &= \left(\frac{1}{2}\right)(2) v_B^2 + 0 \\
\rightarrow \quad & v_B = 5.42 \text{ m/s}
\end{aligned}
$$



Free body diagram

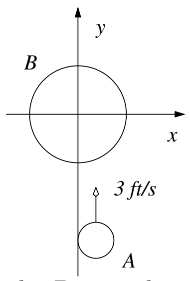
The figure on the left shows the free body diagram of the system when the bob passes through the point B. Here, T denotes the tension in the cord and mg is the weight of the bob. Using the normal-tangent coordinate system Newton's second law, in the normal direction, gives

$$
\begin{aligned}
\sum F_n &= m a_n \\
T - mg &= m \frac{v_B^2}{\rho}.
\end{aligned}
$$

Here, the radius of curvature is $\rho = L = 1.5$ m. Therefore, the tension in the cord at B is

$$T = mg + m \frac{v_B^2}{\rho} = (2)(9.81) + (2) \frac{(5.42)^2}{1.5} = 58.79\text{N}.$$

16. Disk A weights 2 lb and is sliding on a smooth horizontal plane with a velocity of 3 ft/s. Disk B weights 11 lb and is initially at rest. If after the impact A has a velocity of 1 ft/s directed along the positive x axis, determine the speed of B after impact.



Apply the Priniciple of Conservation of Linear Momentum in the x-direction to get

$$
\begin{aligned}
m_A v_{Ax_1} + m_B v_{Bx_1} &= m_A v_{Ax_2} + m_B v_{Bx_2} \\
0 + 0 &= \left(\frac{2}{32.2}\right)(1) + \frac{11}{32.2} v_{Bx_2} \\
\rightarrow \quad & v_{Bx_2} = -0.1818 \text{ ft/s}.
\end{aligned}
$$

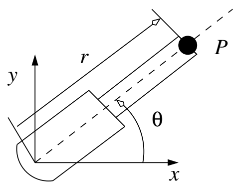
Apply the Priniciple of Conservation of Linear Momentum in the y-direction to get

$$
\begin{aligned}
m_A v_{Ay_1} + m_B v_{By_1} &= m_A v_{Ay_2} + m_B v_{By_2} \\
\left(\frac{2}{32.2}\right)(3) + 0 &= 0 + \frac{11}{32.2} v_{By_2} \\
\rightarrow \quad & v_{By_2} = 0.545 \text{ ft/s}.
\end{aligned}
$$

Therefore the speed of B after impact is

$$|v_{B2}| = \sqrt{(-0.1818)^2 + (0.545)^2} = 0.575 \text{ ft/s}.$$

17. The robot arm is programmed so that point P follows the path $r = 1 - 0.5\cos 2\pi t$ m, $\theta = 0.5 - 0.2\sin 2\pi t$ radians. What is the velocity of P at time $t = 0.8$ s.



Using a polar coordinate system, we have from kinematics that

$$\bar{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$
$$\dot{r} = \frac{dr}{dt} = \pi \sin 2\pi t$$
$$\dot{\theta} = \frac{d\theta}{dt} = -0.4\pi \cos 2\pi t$$

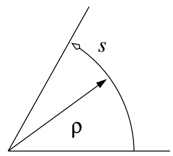
Therefore,

$$\bar{v} = (\pi \sin 2\pi t)\hat{e}_r + (1 - 0.5\cos 2\pi t)(-0.4\pi \cos 2\pi t)\hat{e}_\theta$$

At $t = 0.8$ s, we get

$$\bar{v} = -2.99\,\hat{e}_r - 0.328\,\hat{e}_\theta \text{m/s}.$$

18. A motorcycle starts at $t = 0$ on a circular track of radius $\rho = 400$ m. The tangential component of the acceleration is $a_\tau = 2 + 0.2t$ m/s^2. Determine the magnitude of the acceleration at $t = 10$ s.



Using the normal-tangent coordinate system we have

$$a_\tau = \frac{dv}{dt}$$
$$\int a_\tau dt = v$$
$$\int (2 + 0.2t)\, dt = v$$

Therefore, $v = 2t + 0.1t^2$ m/s. In addition, the acceleration vector is

$$\bar{a} = \frac{v^2}{\rho}\hat{n} + a_\tau \hat{\tau}.$$

At $t = 10$ s we get $v = (2)(10) + 0.1(10^2) = 30$ m/s, $v^2/\rho = (30^2)/400 = 2.25$ m/s^2, and $a_\tau = 2 + (0.2)(10) = 4$ m/s^2. Therefore,

$$\|\bar{a}\| = \sqrt{4^2 + 2.25^2} = 4.59 \text{ m/s}^2.$$